Physics

Determination of the damping ratio

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Purpose of the exercise

The aim of the exercise is to get more familiar with conducting the experiments, collecting the measurements, and correctly processing and analyzing them. The final result is to determine the damping ratio for different surfaces.

Information about QR-Codes

In the margin of the text (on the right) there may be inserts with QR codes. There is no need to scan them, as they are also a clickable link – just click. The QR-code solution is applied after printing the work on physical paper.

This form of additional information has been used to preserve the better visual side of the whole work as well as to make reading smoother by not mixing less important information together with important informations.

1 Description of the task

Description of the task

The aim of the exercise is to study the phenomenon of *Damped Oscillations* and to quantify the damping ratio. The exercise is carried out using phyphox application available for download on the following page: https://phyphox.org/. At the time of writing the report the application is available on Android and IOS smartphones.

I will go back for a moment to very basic information for an introduction and a more accurate description of the phenomenon, in order to bring it closer to the better studied.

The simple harmonic motion was, as the name suggests – simple. In the simplest of assumptions, it was not limited by any external forces, so it could last forever (see Fig. 1).

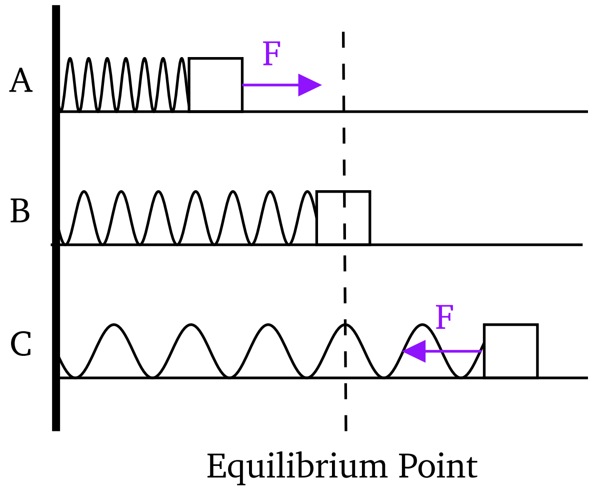


Figure 1: Example of Simple Harmonic Motion and of Simple Harmonic Oscillator

Assuming that there are of course no forces such as, among other things, friction for individual points, we obtain:

A) The spring stretches with the force F opposite to the force pressing on the spring by the box

B) The box is at a point of equilibrium where no force is acting on it, but because of inertia, there is need a force to stop it - which does not exist here and therefore the box exceeds the point of equilibrium.

C) The spring expands until the box stops – when it is on the opposite side of the equilibrium point to a situation in point (A). At this point, the force of the spring wins and takes the box with it, repeating the whole cycle lasting indefinitely.

Of course, this is an extremely ideal situation and it is easy to realise that it is rather unusual in real life. At least one factor can completely ruin it - such as the friction mentioned above – that would stop this movement at some point in time. This is where damped simple harmonic motion, as well as damping oscillations come in.

What we saw in Figure 1 is not just an example of a Simple Harmonic Movement. It is also an example of the Harmonic Oscillator – because it represents not only the movement itself, but also the whole system (spring, box), which just works in the way mentioned above.

1.1 What are damping oscillations?

Damping Oscillations are vibrations whose amplitude decreases with time. This makes us see a certain fading effect – where, for the above example (see Fig. 1), we would observe the box slowing down over time until it stops. We would notice a similar behaviour in the guitar strings.

1.2 What is the damping ratio?

If we already know that our movement will be extinguished by some external force - or in other words, by the loss of energy that is retained in oscillation - then we also know that this ratio exists and that we can find it.
For different values of damping ratio β we can have:

$\beta = 0$	$\beta < 1$	$\beta = 1$	$\beta > 1$
undamped	underdamped	critically damped	overdamped

And for each of these values we see a much different fadeout - often different in terms of not only a small change in amplitude.

2 Measurements and measuring stations

After preparing the measuring place, I moved on to measurements. I dropped the smartphone many times, collecting a stock of measurements, and at the outset I rejected falls where the telephone bounced and ran to the sides – or made pirouettes in the air.

I have also made some measurements for the sake of pure curiosity. I checked the measurements for a very soft pillow and a very hard floor.

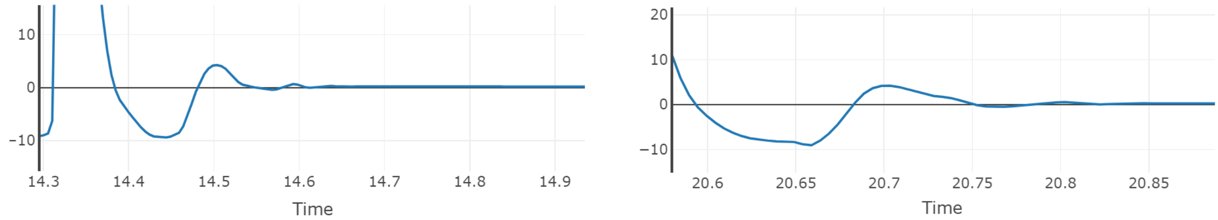


Figure 2: Exemplary drop of smartphone on the pillow

In the picture above we can see a sample drop of a smartphone on a pillow - the fade is practically sudden here and leaves no amplitude/oscillations to measure.

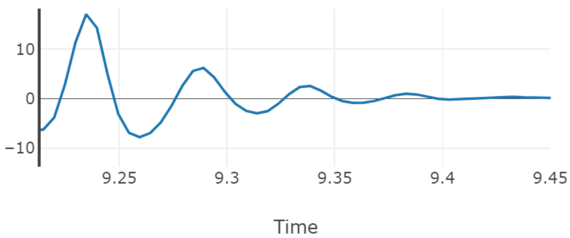
2.1 Bed without blanket

The first measurement was carried out on a bed without any cover. The bed is quite bouncy, so only the tests in which the phone landed as proportionally as possible have been preserved – so in other words, the ones that have been preserved were those in which the hand that threw it off accidentally did not give it such a rotational force that it would bounce back to the sides significantly when falling.

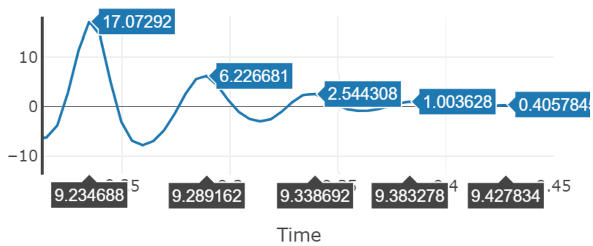
Image

Chair seat with blanket.

2.1.1 First measurement



Graph of the relationship between *absolute acceleration without g* and *time*



Amplitude peaks marked with their occurrence in time

From the above graph I read the following amplitude peaks:

$$a_{max_0} = 17.07292 \longrightarrow t_0 = 9.234688$$
$$a_{max_1} = 6.226681 \longrightarrow t_1 = 9.289162$$
$$a_{max_2} = 2.544308 \longrightarrow t_2 = 9.338692$$
$$a_{max_3} = 1.003628 \longrightarrow t_3 = 9.383278$$
$$a_{max_4} = 0.4057845 \longrightarrow t_4 = 9.427834$$

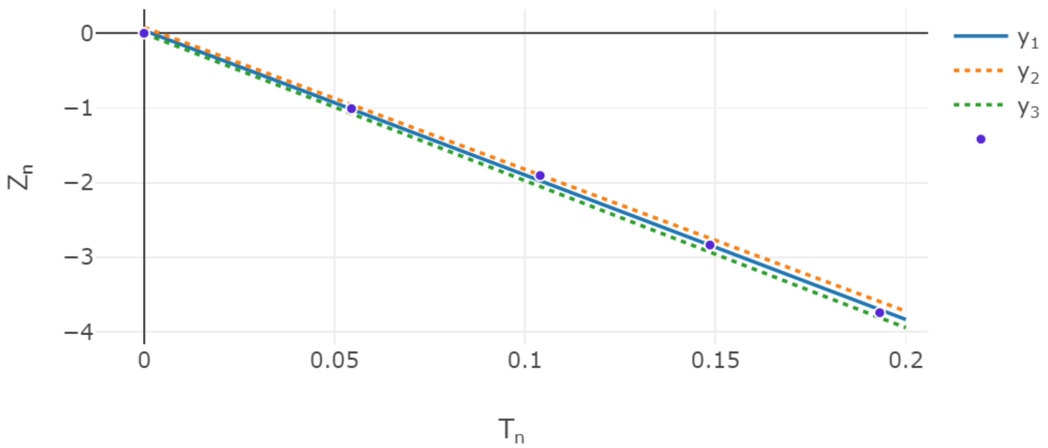
From the above values I get the given T_n and z_n, where $T_n = t_n - t_0$ and $z_n = ln(a_{max_n}/a_{max_0})$.

Tn	Zn
0	0
0.054474	-1.008650136
0.104004	-1.903634875
0.14859	-2.833872148
0.193146	-3.739426631

Using REGLIN.P function I get:

$$y_1 = (-19.34232545)x + (0.037943638)$$
$$y_2 = (-19.34232545 + 0.342305487)x + (0.037943638 + 0.041408669)$$
$$y_3 = (-19.34232545 - 0.342305487)x + (0.037943638 - 0.041408669)$$

My points are (T_n, z_n).



Linear regression of $z_n(T_n)$

I read the value β, which is the absolute value of the directional factor of the linear function, that is:

$$\beta_1 = |-19.34232545| = 19.34232545$$

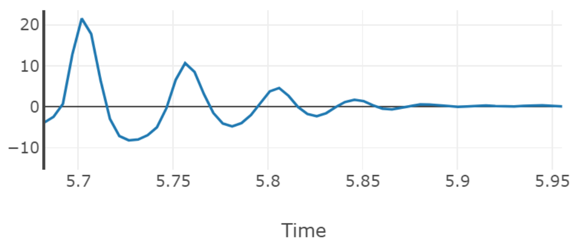
The standard deviation of the adjust is taken as an estimation of u(β).

$$u(\beta_1) = 0.342305487$$

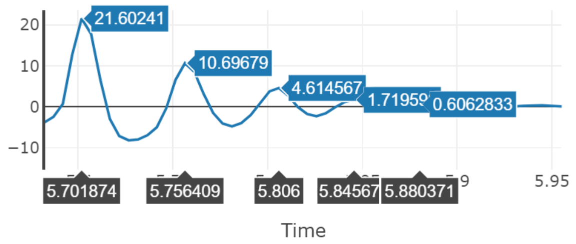
Pearson's correlation coefficient and it's uncertainty:

$r_{xy} = 0.999061307$ $\qquad$ $u(r_{xy}) = 0.052054981$

2.1.2 Second measurement



Graph of the relationship between *absolute acceleration without g* and *time*



Amplitude peaks marked with their occurrence in time

From the above graph I read the following amplitude peaks:

$$a_{max_0} = 21.60241 \longrightarrow t_0 = 5.701874$$
$$a_{max_1} = 10.69679 \longrightarrow t_1 = 5.756409$$
$$a_{max_2} = 4.614567 \longrightarrow t_2 = 5.806$$
$$a_{max_3} = 1.719598 \longrightarrow t_3 = 5.845673$$
$$a_{max_4} = 0.6062833 \longrightarrow t_4 = 5.880371$$

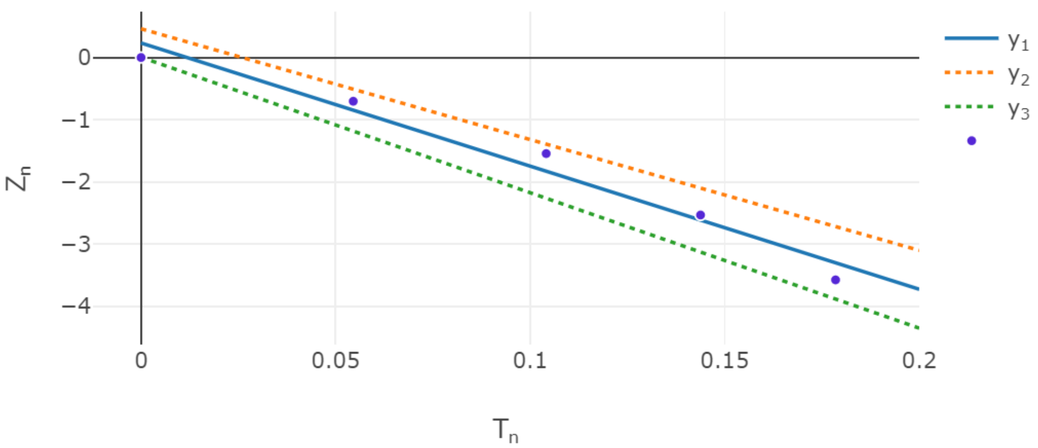
From the above values I get the given T_n and z_n, where $T_n = t_n - t_0$ and $z_n = ln(a_{max_n}/a_{max_0})$.

Tn	Zn
0	0
0.054535	-0.702861186
0.104126	-1.543586843
0.143799	-2.53071434
0.178497	-3.573212793

Using REGLIN.P function I get:

$$y_1 = (-19.78657099)x + (0.233222932)$$
$$y_2 = (-19.78657099 + 1.983835945)x + (0.233222932 + 0.228538627)$$
$$y_3 = (-19.78657099 - 1.983835945)x + (0.233222932 - 0.228538627)$$

My points are (T_n, z_n).



Linear regression of $z_n(T_n)$

I read the value β, which is the absolute value of the directional factor of the linear function, that is:

$$\beta_2 = |-19.78657099| = 19.78657099$$

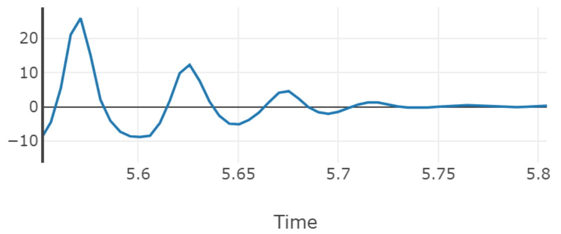
The standard deviation of the adjust is taken as an estimation of u(β).

$$u(\beta_2) = 1.983835945$$

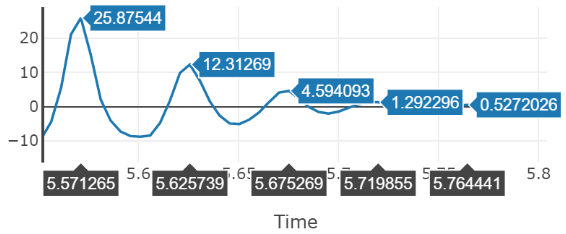
Pearson's correlation coefficient and it's uncertainty:

$$r_{xy} = 0.970725589 \qquad u(r_{xy}) = 0.281199171$$

2.1.3 Third measurement



Graph of the relationship between *absolute acceleration without g* and *time*



Amplitude peaks marked with their occurrence in time

From the above graph I read the following amplitude peaks:

$$a_{max_0} = 25.87544 \longrightarrow t_0 = 5.571265$$
$$a_{max_1} = 12.31269 \longrightarrow t_1 = 5.625739$$
$$a_{max_2} = 4.594093 \longrightarrow t_2 = 5.675269$$
$$a_{max_3} = 1.292296 \longrightarrow t_3 = 5.719855$$
$$a_{max_4} = 0.5272026 \longrightarrow t_4 = 5.764441$$

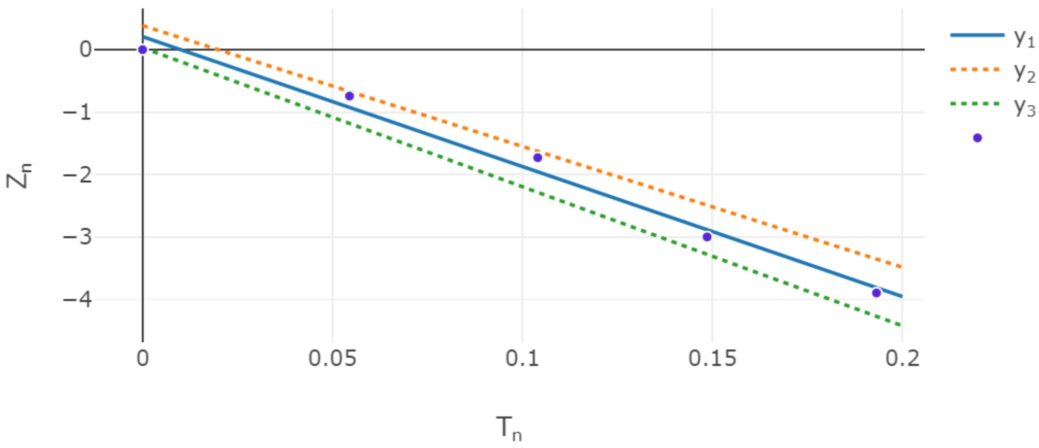
From the above values I get the given T_n and z_n, where $T_n = t_n - t_0$ and $z_n = ln(a_{max_n}/a_{max_0})$.

Tn	Zn
0	0
0.054474	-0.742663818
0.104004	-1.728522908
0.14859	-2.996873775
0.193176	-3.89346462

Using REGLIN.P function I get:

$$y_1 = (-20.79637963)x + (0.208347802)$$
$$y_2 = (-20.79637963 + 1.460133731)x + (0.208347802 + 0.176646257)$$
$$y_3 = (-20.79637963 - 1.460133731)x + (0.208347802 - 0.176646257)$$

My points are (T_n, z_n).



Linear regression of $z_n(T_n)$

I read the value β, which is the absolute value of the directional factor of the linear function, that is:

$$\beta_3 = |-20.79637963| = 20.79637963$$

The standard deviation of the adjust is taken as an estimation of u(β).

$$u(\beta_3) = 1.460133731$$

Pearson's correlation coefficient and it's uncertainty:

$$r_{xy} = 0.985426784 \qquad u(r_{xy}) = 0.222071853$$

2.1.4 Weighted average of the damping ratio

Then I calculate the weighted average from the above calculated damping ratio:

$$\overline{x_w} = \frac{\sum_{i=1}^{n} x_i\, w_i}{\sum_{i=1}^{n} w_i} = \frac{\left(19.34232545 \cdot \frac{1}{(0.342305487)^2}\right) + \left(19.78657099 \cdot \frac{1}{(1.983835945)^2}\right) + \left(20.79637963 \cdot \frac{1}{(1.460133731)^2}\right)}{\frac{1}{(0.342305487)^2} + \frac{1}{(1.983835945)^2} + \frac{1}{(1.460133731)^2}}$$

$$\approx 19.428190$$

Finally, I calculate the uncertainty of the weighted average:

$$u\left(\overline{x_w}\right) = \frac{1}{\sqrt{\sum_{i=1}^{n} w_i}} = \frac{1}{\sqrt{\frac{1}{(0.342305487)^2} + \frac{1}{(1.983835945)^2} + \frac{1}{(1.460133731)^2}}} \approx 0.3286643$$

2.1.5 Final measurement result for a bed without a blanket

We can write the final result in three ways:

A) The final damping ratio β measures 19.43 with an uncertainty of 0.33

B) β = 19.43; U(β) = 0.33.

C) β = 19.43(33).

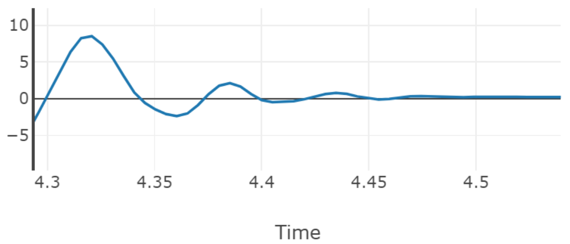
2.2 Bed with blanket

Another measurement was made on a bed covered with a blanket. The oscillations in this case should disappear faster than on a bed without a blanket. This will be an interesting comparison to the previous example. This measurement was definitely easier because the phone did not bounce so much, nor did it bounce so much to the sides.

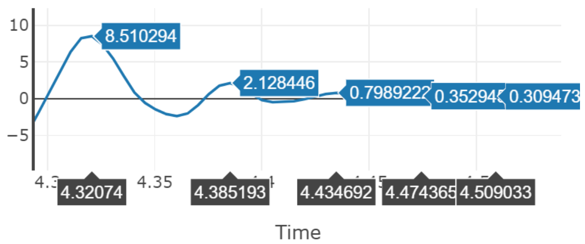
Image

Chair seat with blanket.

2.2.1 First measurement



Graph of the relationship between *absolute acceleration without g* and *time*



Amplitude peaks marked with their occurrence in time

From the above graph I read the following amplitude peaks:

$$a_{max_0} = 8.511029 \longrightarrow t_0 = 4.32074$$
$$a_{max_1} = 2.128446 \longrightarrow t_1 = 4.385193$$
$$a_{max_2} = 0.7989222 \longrightarrow t_2 = 4.434692$$
$$a_{max_3} = 0.3529451 \longrightarrow t_3 = 4.474365$$
$$a_{max_4} = 0.309473 \longrightarrow t_4 = 4.509033$$

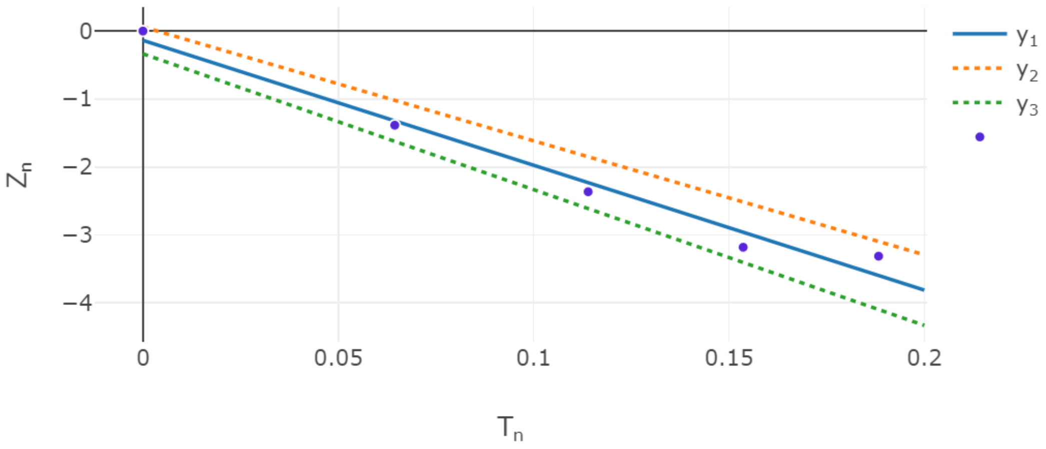
From the above values I get the given T_n and z_n, where $T_n = t_n - t_0$ and $z_n = ln(a_{max_n}/a_{max_0})$.

Tn	Zn
0	0
0.064453	-1.38597
0.113952	-2.36585
0.153625	-3.18281
0.188293	-3.31425

Using REGLIN.P function I get:

$$y_1 = (-18.38742952)x + (-0.136295173)$$
$$y_2 = (-18.38742952 + 1.606753749)x + (-0.136295173 + 0.19834676)$$
$$y_3 = (-18.38742952 - 1.606753749)x + (-0.136295173 - 0.19834676)$$

My points are (T_n, z_n).



Linear regression of $z_n(T_n)$

I read the value β, which is the absolute value of the directional factor of the linear function, that is:

$$\beta_1 = |-18.38742952| = 18.38742952$$

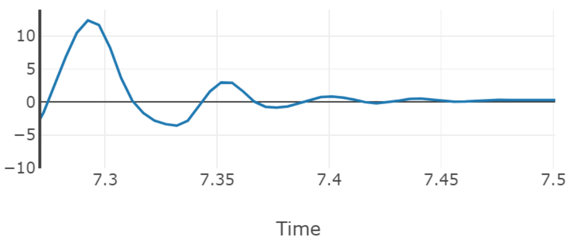
The standard deviation of the adjust is taken as an estimation of u(β).

$$u(\beta_1) = 1.606753749$$

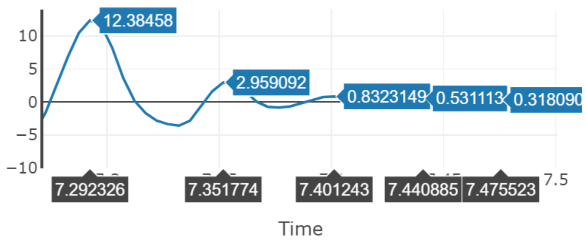
Pearson's correlation coefficient and it's uncertainty:

$$r_{xy} = 0.977605494 \qquad u(r_{xy}) = 0.238574188$$

2.2.2 Second measurement



Graph of the relationship between *absolute acceleration without g* and *time*



Amplitude peaks marked with their occurrence in time

From the above graph I read the following amplitude peaks:

$$a_{max_0} = 12.38458 \longrightarrow t_0 = 7.292326$$
$$a_{max_1} = 2.959092 \longrightarrow t_1 = 7.351774$$
$$a_{max_2} = 0.8323149 \longrightarrow t_2 = 7.401243$$
$$a_{max_3} = 0.531113 \longrightarrow t_3 = 7.440885$$
$$a_{max_4} = 0.3180904 \longrightarrow t_4 = 7.475523$$

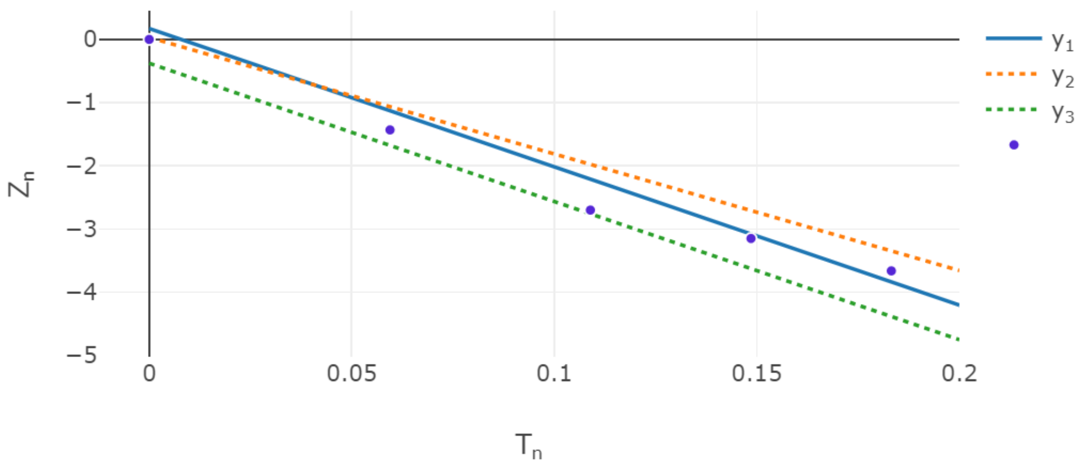
From the above values I get the given T_n and z_n, where $T_n = t_n - t_0$ and $z_n = ln(a_{max_n}/a_{max_0})$.

Tn	Zn
0	0
0.059448	-1.431569686
0.108917	-2.699996575
0.148559	-3.149232625
0.183197	-3.66187181

Using REGLIN.P function I get:

$$y_1 = (-20.15542695)x + (-0.172503682)$$
$$y_2 = (-20.15542695 + 1.715187994)x + (-0.172503682 + 0.204428385)$$
$$y_3 = (-20.15542695 - 1.715187994)x + (-0.172503682 - 0.204428385)$$

My points are (T_n, z_n).



Linear regression of $z_n(T_n)$

I read the value β, which is the absolute value of the directional factor of the linear function, that is:

$$\beta_2 = |-20.15542695| = 20.15542695$$

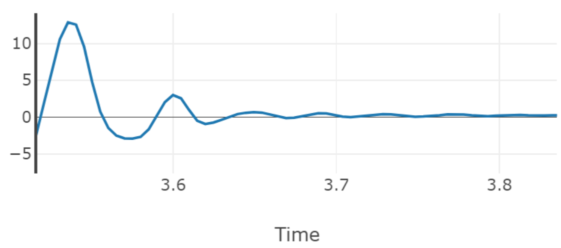
The standard deviation of the adjust is taken as an estimation of u(β).

$$u(\beta_2) = 1.715187994$$

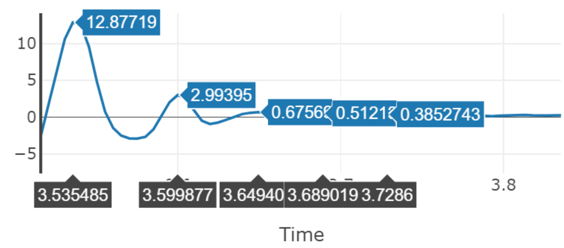
Pearson's correlation coefficient and it's uncertainty:

$$r_{xy} = 0.978736896 \qquad u(r_{xy}) = 0.248576202$$

2.2.3 Third measurement



Graph of the relationship between *absolute acceleration without g* and *time*



Amplitude peaks marked with their occurrence in time

From the above graph I read the following amplitude peaks:

$$
\begin{aligned}
a_{max_0} &= 12.87719 & \longrightarrow \quad t_0 &= 3.535485 \\
a_{max_1} &= 2.99395 & \longrightarrow \quad t_1 &= 3.599877 \\
a_{max_2} &= 0.6756997 & \longrightarrow \quad t_2 &= 3.649407 \\
a_{max_3} &= 0.5121306 & \longrightarrow \quad t_3 &= 3.689019 \\
a_{max_4} &= 0.3852743 & \longrightarrow \quad t_4 &= 3.7286
\end{aligned}
$$

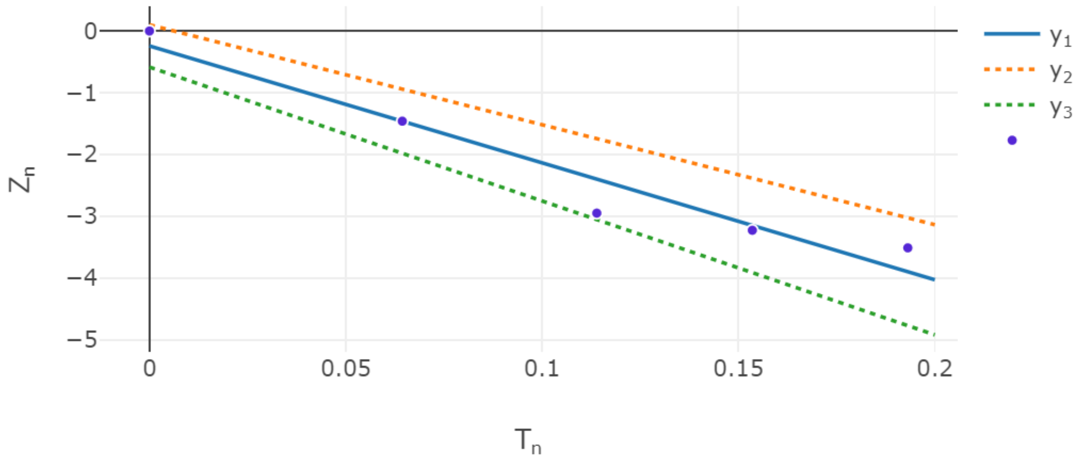
From the above values I get the given T_n and z_n, where $T_n = t_n - t_0$ and $z_n = ln(a_{max_n}/a_{max_0})$.

Tn	Zn
0	0
0.064392	-1.458863943
0.113922	-2.947464062
0.153534	-3.224633138
0.193115	-3.50925726

Using REGLIN.P function I get:

$$
\begin{aligned}
y_1 &= (-18.9241849)x + (-0.241144305) \\
y_2 &= (-18.9241849 + 2.743643689)x + (-0.241144305 + 0.342660434) \\
y_3 &= (-18.9241849 - 2.743643689)x + (-0.241144305 - 0.342660434)
\end{aligned}
$$

My points are (T_n, z_n).



Linear regression of $z_n(T_n)$

I read the value β, which is the absolute value of the directional factor of the linear function, that is:

$$\beta_3 = |-18.9241849| = 18.9241849$$

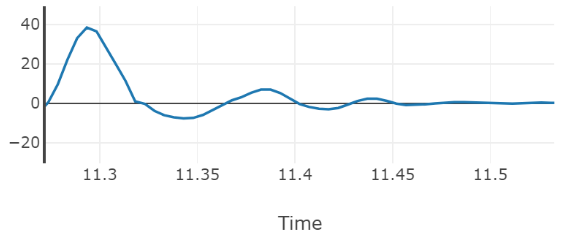
The standard deviation of the adjust is taken as an estimation of u(β).

$$u(\beta_3) = 2.743643689$$

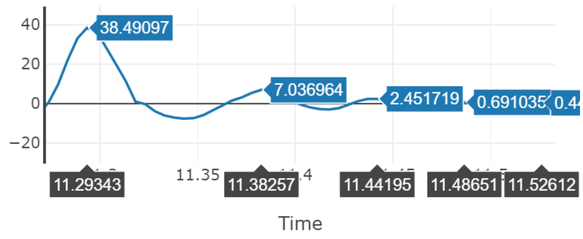
Pearson's correlation coefficient and it's uncertainty:

$$r_{xy} = 0.940682174 \qquad u(r_{xy}) = 0.414947549$$

2.2.4 Fourth measurement



Graph of the relationship between *absolute acceleration without g* and *time*



Amplitude peaks marked with their occurrence in time

From the above graph I read the following amplitude peaks:

$$a_{max_0} = 38.49097 \longrightarrow t_0 = 11.29343$$
$$a_{max_1} = 7.036964 \longrightarrow t_1 = 11.38257$$
$$a_{max_2} = 2.451719 \longrightarrow t_2 = 11.44195$$
$$a_{max_3} = 0.6910355 \longrightarrow t_3 = 11.48651$$
$$a_{max_4} = 0.4451219 \longrightarrow t_4 = 11.52612$$

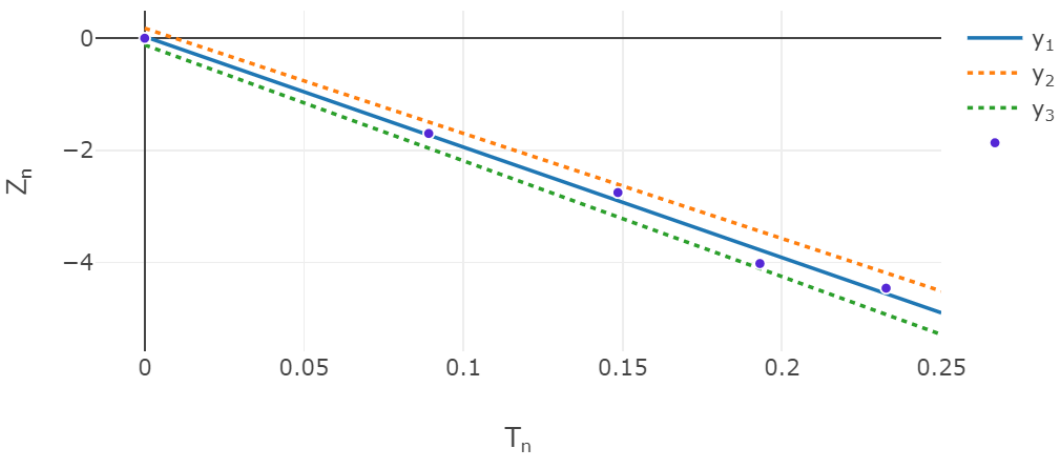
From the above values I get the given T_n and z_n, where $T_n = t_n - t_0$ and $z_n = ln(a_{max_n}/a_{max_0})$.

Tn	Zn
0	0
0.08914	-1.699246841
0.14852	-2.753634257
0.19308	-4.01998775
0.23269	-4.45983077

Using REGLIN.P function I get:

$$y_1 = (-19.71180637)x + (0.028940816)$$
$$y_2 = (-19.71180637 + 0.954750667)x + (0.028940816 + 0.148787195)$$
$$y_3 = (-19.71180637 - 0.954750667)x + (0.028940816 - 0.148787195)$$

My points are (T_n, z_n).



Linear regression of $z_n(T_n)$

I read the value β, which is the absolute value of the directional factor of the linear function, that is:

$$\beta_4 = |-19.71180637| = 19.71180637$$

The standard deviation of the adjust is taken as an estimation of u(β).

$$u(\beta_4) = 0.954750667$$

Pearson's correlation coefficient and it's uncertainty:

$$r_{xy} = 0.993011202 \qquad u(r_{xy}) = 0.174489088$$

2.2.5 Weighted average of the damping ratio

Then I calculate the weighted average from the above calculated damping ratio:

$$\overline{x_w} = \frac{\sum\limits_{i=1}^{n} x_i\, w_i}{\sum\limits_{i=1}^{n} w_i} = \frac{\begin{gathered}\left(18.38742952 \cdot \frac{1}{(1.606753749)^2}\right) + \left(20.15542695 \cdot \frac{1}{(1.715187994)^2}\right) \\ + \left(18.9241849 \cdot \frac{1}{(2.743643689)^2}\right) + \left(19.71180637 \cdot \frac{1}{(0.954750667)^2}\right)\end{gathered}}{\frac{1}{(1.606753749)^2} + \frac{1}{(1.715187994)^2} + \frac{1}{(2.743643689)^2} + \frac{1}{(0.954750667)^2}} \approx 19.47328$$

Finally, I calculate the uncertainty of the weighted average:

$$u\left(\overline{x_w}\right) = \frac{1}{\sqrt{\sum\limits_{i=1}^{n} w_i}} = \frac{1}{\sqrt{\frac{1}{(1.606753749)^2} + \frac{1}{(1.715187994)^2} + \frac{1}{(2.743643689)^2} + \frac{1}{(0.954750667)^2}}} = 0.7148$$

2.2.6 Final measurement result for a bed with a blanket

We can write the final result in three ways:

A) The final damping ratio β measures 19.47 with an uncertainty of 0.71

B) β = 19.47; U(β) = 0.71.

C) β = 19.47(71).

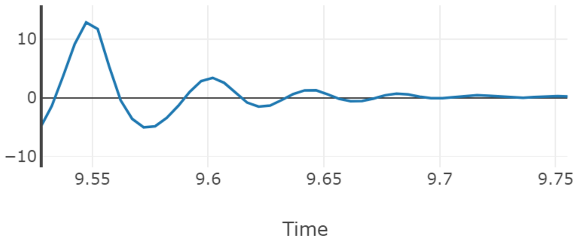
2.3 Chair seat with blanket

This design is essentially very simple. Again, a suitable layer of blanket is applied to the quite bouncy seat of the office chair. The seat is relatively soft - it resembles foam and is rather flat. Initially there were attempts to measure on the chair itself, but the smartphone was dangerously fond of bouncing to the sides and could not stabilise when falling - that is why the chair was covered with a blanket.

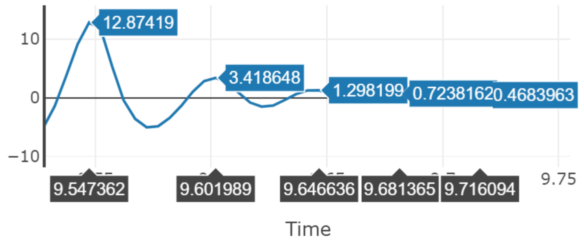
Image

Chair seat with blanket.

2.3.1 First measurement



Graph of the relationship between *absolute acceleration without g* and *time*



Amplitude peaks marked with their occurrence in time

From the above graph I read the following amplitude peaks:

$$a_{max_0} = 12.87419 \longrightarrow t_0 = 9.547362$$
$$a_{max_1} = 3.418648 \longrightarrow t_1 = 9.601989$$
$$a_{max_2} = 1.298199 \longrightarrow t_2 = 9.646636$$
$$a_{max_3} = 0.7238162 \longrightarrow t_3 = 9.681365$$
$$a_{max_4} = 0.4683963 \longrightarrow t_4 = 9.716094$$

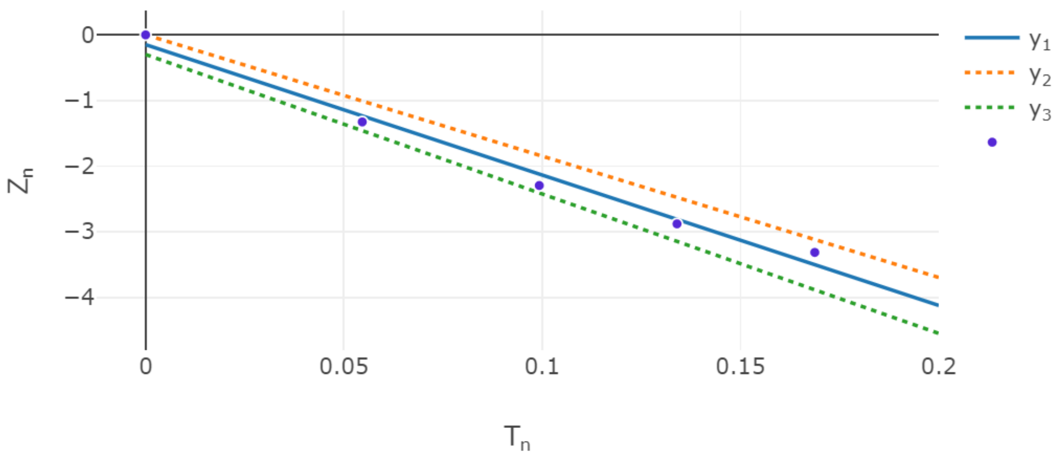
From the above values I get the given T_n and z_n, where $T_n = t_n - t_0$ and $z_n = ln(a_{max_n}/a_{max_0})$.

Tn	Zn
0	0
0.054627	-1.325979381
0.099274	-2.294246613
0.134003	-2.878442318
0.168732	-3.313665078

Using REGLIN.P function I get:

$$y_1 = (-19.86522043)x + (-0.148231718)$$
$$y_2 = (-19.86522043) + 1.379084312)x + (-0.148231718 + 0.150145144)$$
$$y_3 = (-19.86522043) - 1.379084312)x + (-0.148231718 - 0.150145144)$$

My points are (T_n, z_n).



Linear regression of $z_n(T_n)$

I read the value β, which is the absolute value of the directional factor of the linear function, that is:

$$\beta_1 = |-19.86522043| = 19.86522043$$

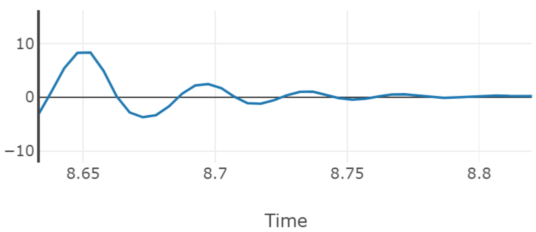
The standard deviation of the adjust is taken as an estimation of u(β).

$$u(\beta_1) = 1.379084312$$

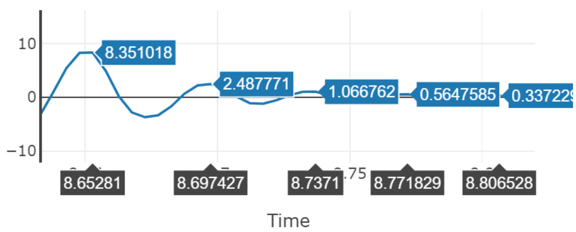
Pearson's correlation coefficient and it's uncertainty:

$$r_{xy} = 0.985747799 \qquad u(r_{xy}) = 0.182766089$$

2.3.2 Second measurement



Graph of the relationship between *absolute acceleration without g* and *time*



Amplitude peaks marked with their occurrence in time

From the above graph I read the following amplitude peaks:

$$\begin{aligned}
a_{max_0} &= 8.351018 &\longrightarrow\quad t_0 &= 8.65281\\
a_{max_1} &= 2.487771 &\longrightarrow\quad t_1 &= 8.697427\\
a_{max_2} &= 1.066762 &\longrightarrow\quad t_2 &= 8.7371\\
a_{max_3} &= 0.5647585 &\longrightarrow\quad t_3 &= 8.771829\\
a_{max_4} &= 0.3372291 &\longrightarrow\quad t_4 &= 8.806528
\end{aligned}$$

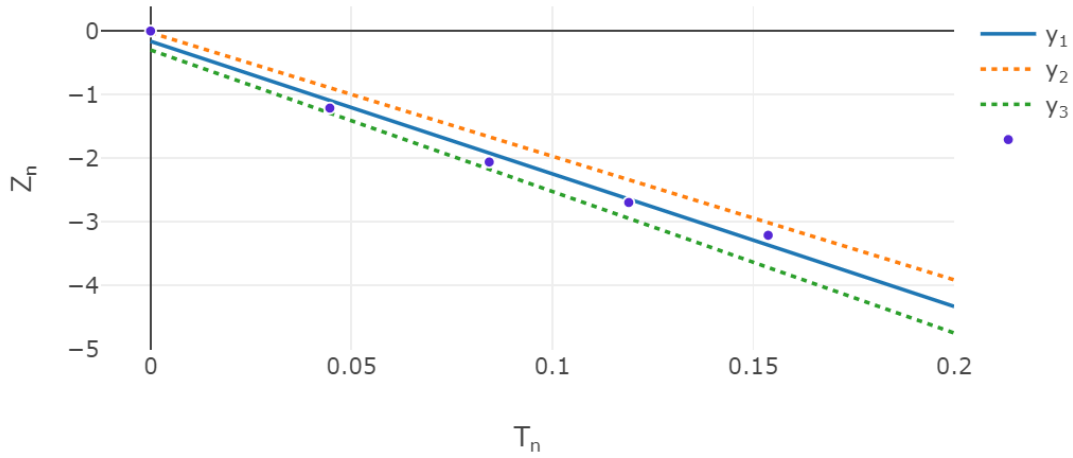
From the above values I get the given T_n and z_n, where $T_n = t_n - t_0$ and $z_n = ln(a_{max_n}/a_{max_0})$.

Tn	Zn
0	0
0.044617	-1.210996319
0.08429	-2.057755555
0.119019	-2.69374052
0.153718	-3.209376205

Using REGLIN.P function I get:

$$y_1 = (-20.82905623)x + (-0.161200628)$$
$$y_2 = (-20.82905623 + 1.406495361)x + (-0.161200628 + 0.136205445)$$
$$y_3 = (-20.82905623 - 1.406495361)x + (-0.161200628 - 0.136205445)$$

My points are (T_n, z_n).



Linear regression of $z_n(T_n)$

I read the value β, which is the absolute value of the directional factor of the linear function, that is:

$$\beta_2 = |-20.82905623| = 20.82905623$$

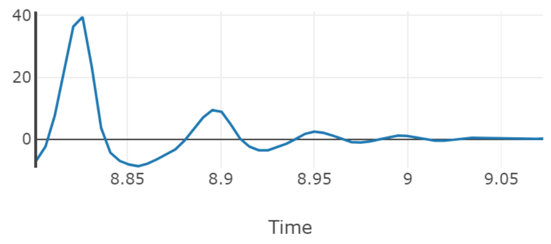
The standard deviation of the adjust is taken as an estimation of u(β).

$$u(\beta_2) = 1.406495361$$

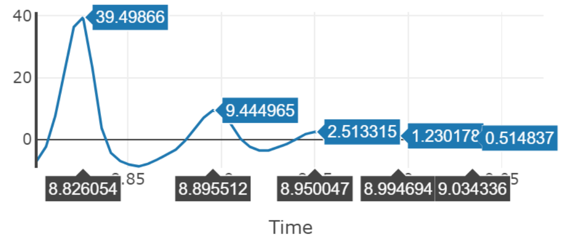
Pearson's correlation coefficient and it's uncertainty:

$$r_{xy} = 0.986505457 \qquad u(r_{xy}) = 0.170102494$$

2.3.3 Third measurement



Graph of the relationship between *absolute acceleration without g* and *time*



Amplitude peaks marked with their occurrence in time

From the above graph I read the following amplitude peaks:

$$a_{max_0} = 39.49866 \longrightarrow t_0 = 8.826054$$
$$a_{max_1} = 9.444965 \longrightarrow t_1 = 8.895512$$
$$a_{max_2} = 2.513315 \longrightarrow t_2 = 8.950047$$
$$a_{max_3} = 1.230178 \longrightarrow t_3 = 8.994694$$
$$a_{max_4} = 0.514837 \longrightarrow t_4 = 9.034336$$

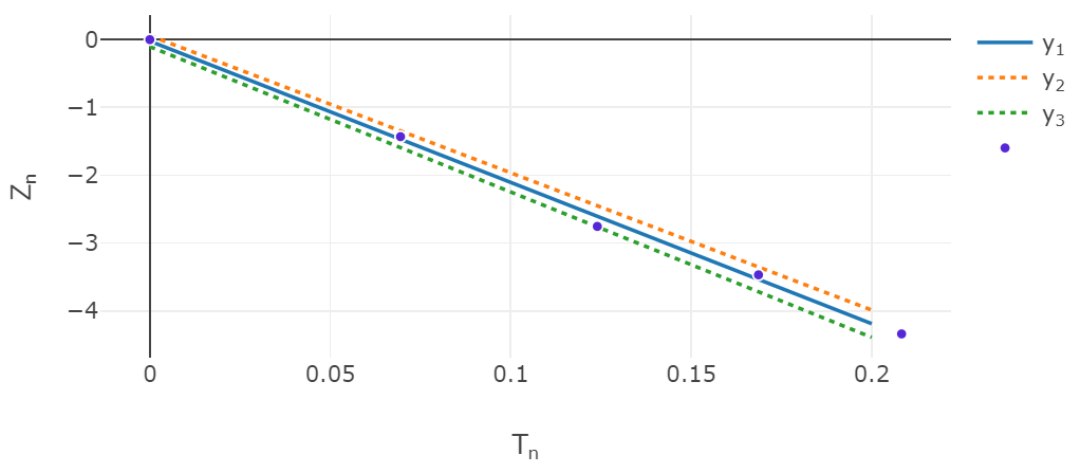
From the above values I get the given T_n and z_n, where $T_n = t_n - t_0$ and $z_n = ln(a_{max_n}/a_{max_0})$.

Tn	Zn
0	0
0.069458	-1.430784952
0.123993	-2.754664148
0.16864	-3.469107873
0.208282	-4.340171681

Using REGLIN.P function I get:

$$y_1 = (-20.83717937)x + (-0.021952829)$$
$$y_2 = (-20.83717937 + 0.600074491)x + (-0.021952829 + 0.081406699)$$
$$y_3 = (-20.83717937 - 0.600074491)x + (-0.021952829 - 0.081406699)$$

My points are (T_n, z_n).



Linear regression of $z_n(T_n)$

I read the value β, which is the absolute value of the directional factor of the linear function, that is:

$$\beta_3 = |-20.83717937| = 20.83717937$$

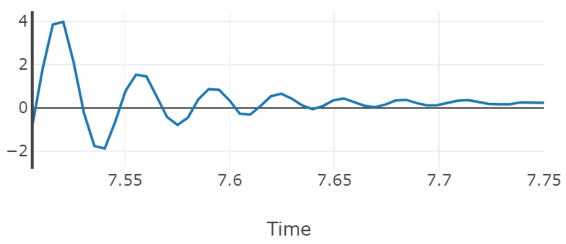
The standard deviation of the adjust is taken as an estimation of u(β).

$$u(\beta_3) = 0.600074491$$

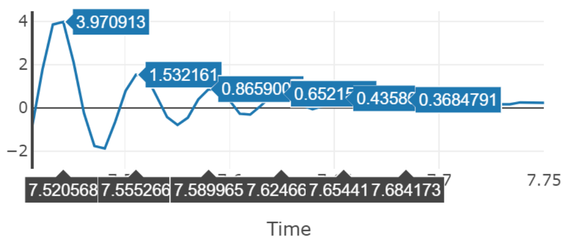
Pearson's correlation coefficient and it's uncertainty:

$$r_{xy} = 0.997518156 \qquad u(r_{xy}) = 0.098519087$$

2.3.4 Fourth measurement



Graph of the relationship between *absolute acceleration without g* and *time*



Amplitude peaks marked with their occurrence in time

From the above graph I read the following amplitude peaks:

$$a_{max_0} = 3.970913 \longrightarrow t_0 = 7.520568$$
$$a_{max_1} = 1.532161 \longrightarrow t_1 = 7.555266$$
$$a_{max_2} = 0.8659002 \longrightarrow t_2 = 7.589965$$
$$a_{max_3} = 0.6521568 \longrightarrow t_3 = 7.624663$$
$$a_{max_4} = 0.4358637 \longrightarrow t_4 = 7.654418$$
$$a_{max_5} = 0.3684791 \longrightarrow t_5 = 7.684173$$

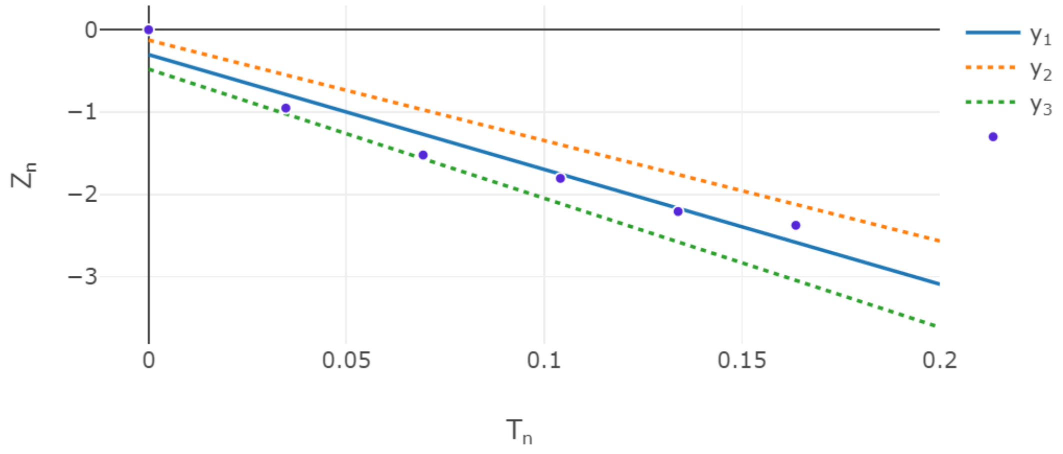
From the above values I get the given T_n and z_n, where $T_n = t_n - t_0$ and $z_n = ln(a_{max_n}/a_{max_0})$.

Tn	Zn
0	0
0.034698	-0.952316886
0.069397	-1.522981663
0.104095	-1.806466298
0.13385	-2.209421742
0.163605	-2.377367328

Using REGLIN.P function I get:

$$y_1 = (-13.95320813)x + (-0.302197332)$$
$$y_2 = (-13.95320813 + 1.742987271)x + (-0.302197332 + 0.176518639)$$
$$y_3 = (-13.95320813 - 1.742987271)x + (-0.302197332 - 0.176518639)$$

My points are (T_n, z_n).



Linear regression of $z_n(T_n)$

I read the value β, which is the absolute value of the directional factor of the linear function, that is:

$$\beta_4 = |-13.95320813| = 13.95320813$$

The standard deviation of the adjust is taken as an estimation of u(β).

$$u(\beta_4) = 1.742987271$$

Pearson's correlation coefficient and it's uncertainty:

$$r_{xy} = 0.941250332 \qquad u(r_{xy}) = 0.239781279$$

2.3.5 Weighted average of the damping ratio

Then I calculate the weighted average from the above calculated damping ratio:

$$\overline{x_w} = \frac{\sum_{i=1}^{n} x_i\, w_i}{\sum_{i=1}^{n} w_i} = \frac{\left(19.86522043 \cdot \frac{1}{(1.379084312)^2}\right) + \left(20.82905623 \cdot \frac{1}{(1.406495361)^2}\right) + \left(20.83717937 \cdot \frac{1}{(0.600074491)^2}\right) + \left(13.95320813 \cdot \frac{1}{(1.742987271)^2}\right)}{\frac{1}{(1.379084312)^2} + \frac{1}{(1.406495361)^2} + \frac{1}{(0.600074491)^2} + \frac{1}{(1.742987271)^2}} \approx 20.1650$$

Finally, I calculate the uncertainty of the weighted average:

$$u\left(\overline{x_w}\right) = \frac{1}{\sqrt{\sum_{i=1}^{n} w_i}} = \frac{1}{\sqrt{\frac{1}{(1.379084312)^2} + \frac{1}{(1.406495361)^2} + \frac{1}{(0.600074491)^2} + \frac{1}{(1.742987271)^2}}} \approx 0.4916$$

2.3.6 Final measurement result for chair seat with a blanket

We can write the final result in three ways:

A) The final damping ratio β measures 20.17 with an uncertainty of 0.49

B) β = 20.17; U(β) = 0.49.

C) β = 20.17(49).

3 Conclusions

It was a very interesting experience to look for different surfaces and to notice that proper "damping" of the surface significantly influences how quickly the amplitude disappears. Unfortunately, I did not have many surfaces or resilient mattresses. By far the most difficult thing was the later part related to drawing diagrams. I found many questions there and I stopped for a very long time not knowing how to deal with many things.

References

[1] Jeff Sanny, Loyola Marymount University, Samuel Ling, and Truman State University. *University Physics*, volume 1, chapter 15.5 Damped Oscillations. OpenStax, 9 2020. https://openstax.org/details/books/university-physics-volume-1.

[2] Steven Holzner. *Physics for Dummies*, chapter 12 Springs-n-Things: Simple Harmonic Motion. Wiley Publishing, Inc, 2006.

[3] Wikipedia contributors. Damping ratio — Wikipedia, the free encyclopedia. https://en.wikipedia.org/w/index.php?title=Damping_ratio&oldid=988118022, 2020. [Online; accessed 3-December-2020].