Physics

Determining the speed of sound in the air

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Purpose of the exercise

The aim of this task is to get to know more about the proper performance of measurements and related issues. The final result is to calculate the speed of sound propagation in the air that is close to the real value. The learning is to be based on mistakes, drawing conclusions and repeating experiences.

1 Description of the task

Description of the task

The aim of the exercise is to experimentally determine the speed of sound propagation in the air. Determination of this parameter is done using smartphones or tablets with an installed application in which an acoustic stopwatch is used. Thanks to the possibility of measuring the difference in time of reaching the devices located at different distances from its source, the following is calculated the speed of sound.

The exercise is carried out using phyphox application available for download on the following page: https://phyphox.org/. At the time of writing the report the application is available on Android and IOS smartphones.

1.1 What are sound waves?

Sound waves are different pressure variations in a medium such as air – these are respectively compression (high pressure regions) and decompression (low pressure regions). The corresponding vibrations of our vocal cords, among other things, in their own way "push" the molecules that are pushing the adjacent (guilty of nothing) molecules.

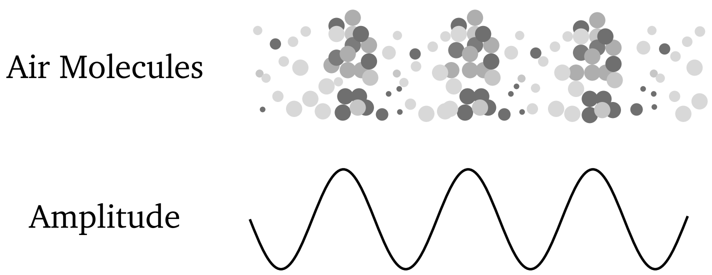


Figure 1: How we can imagine the sound wave, based on informations provided [1]

As we are going to deal specifically with sound in the air, imagine that the air consists of small air particles. These molecules, when pushed, start to press on the adjacent molecules and, importantly, it is not the air that moves further. It does not go too far, but what really "moves" is compression. So in a way, the sound wave is some kind of pressure disturbance that travels through a medium due to particle-to-particle interactions.

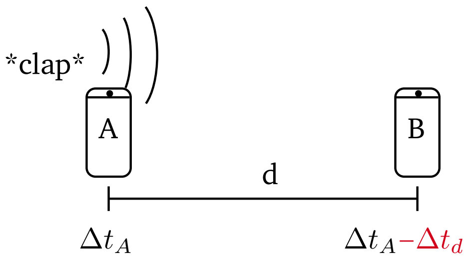
1.2 Why it is temperature-dependent?

Heat – exactly how sound is a form of kinetic energy. Molecules at higher temperatures have more energy, so they vibrate faster. Using the above description (*Section 1.1. What are sound waves?*), we can say that thanks to this, molecules "push" themselves faster (this is called elastic property).

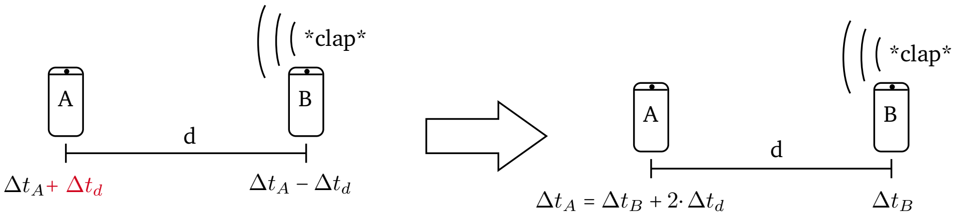
1.3 Why it is humidity-dependent?

The simple question – the simple answer, water vapor in the air affects the density of the air mass (we call this inertial property).

2 Measurments



If we assume that when the person on device A claps and the acoustic stopwatch in device A starts measuring the time – let's give it t_A, then due to the delay, which is the speed of sound, the screen of device B will show the time $t_A - \Delta t_d$ where Δt_d is the difference in time resulting from the speed of sound.



Now the person on device B claps what stops the time measured on device B screen and finally it is equal to $t_A - \Delta t_d$. But again, the sound must reach device A, which extends the time measured by device t_A by Δt_d. Finally, the time measured by device A is $t_A + \Delta t_d$. In other words – device time A indicates device time B plus double the delay time that occurred and delayed device counter B twice.

If we now take advantage of the dependence that speed is the distance travelled over time, we will it:

$$v = \frac{distance}{time} = \frac{2d}{|\Delta t_A - \Delta t_B|} \tag{1}$$

2.1 House

The first measurement was made inside the house, where the temperature was $24.1°C$ (the measurement was carried out in a closed room), the humidity was unknown and the measurements were made at a distance of 5 meters.

First device recording [s]	Second device recording [s]	Δt [s]
0.941	0.909	0.032
0.758	0.726	0.032
1.18	1.155	0.025
0.889	0.855	0.034
0.934	0.905	0.029
0.941	0.909	0.032
0.634	0.608	0.026
0.571	0.542	0.029
0.94	0.907	0.033
1.151	1.125	0.026
0.853	0.826	0.027

Figure 2: Measurement of the "claps" timings

Temperature $°C$
24.1
24.1
24.1

Figure 3: Measurement of the temperature

2.1.1 The average value and its uncertainty

From the given data I calculate the average value:

$$\bar{t} = \frac{1}{n}\sum_{i=1}^{n} x_i = \frac{0.032\ +\ 0.032\ +\ 0.025\ +\ ...\ +\ 0.033\ +\ 0.026\ +\ 0.027\ [s]}{11} = 0.029545455\ s$$

From the given data I also calculate the uncertainty of average values:

$$\sigma_{\overline{x}} = \frac{\sigma}{\sqrt{n}} = \sqrt{\frac{1}{n(n-1)}\sum_{i=1}^{n}(t_i - \bar{t})^2} \approx 0.000966377\ s, \quad \text{additionaly: } \sigma = 0.00320511\ s$$

2.1.2 Calculation of sound speed in the air

Using the equation **(1)**:

$$V = \frac{2D}{\Delta t} = \frac{2 \cdot 5\ [m]}{0.029545455\ [s]} \approx 338.4615\ \frac{m}{s}$$

Info: We can see that the result is fairly close, although still far from being a precise answer. For a temperature of $24.1°C$, an assumed air pressure of 101.325 kPa, and an assumed room humidity of 40%, the sound velocity should be: *346.4* $\frac{m}{s}$. Comparative data was taken from the online calculator [2].

2.1.3 Calculation of the uncertainty of the determined speed

Using the law of propagation of uncertainty for the function of many variables I use a formula:

$$u_c(y) = \sqrt{\sum_k \left[\frac{\partial y}{\partial x_k} u(x_k)\right]^2}$$

I consider the measuring uncertainty of the measuring instrument to be the smallest measuring range – 1mm in my case. My measuring tape measured 5 meters and was used to measure 5 meters of distance. In short, the measured distance is 5 meters with an uncertainty of 0.001 meters. Using those informations I calculate:

$$u_c(V) = \sqrt{[u(D)]^2 + [u(t)]^2} = \sqrt{\left[\frac{\partial V}{\partial D}\left(\frac{2D}{t}\right)u(D)\right]^2 + \left[\frac{\partial V}{\partial t}\left(\frac{2D}{t}\right)u(t)\right]^2} = \sqrt{\left[\left(\frac{2}{t}\right)u(D)\right]^2 + \left[\left(-\frac{2d}{t^2}\right)u(t)\right]^2} =$$

$$= \sqrt{\left[\left(\frac{2}{0.029545455\ s}\right)\cdot 0.001m\right]^2 + \left[\left(-\frac{2\cdot 5\ [m]}{(0.029545455\ s)^2}\right)\cdot 0.000966377\ s\right]^2} = 11.07\ \frac{m}{s}$$

2.1.4 The resulting sound speed value

A) The calculated speed measures 338.46 $\frac{m}{s}$ with an uncertainty of 11.07 $\frac{m}{s}$.

B) V = 338.46 $\frac{m}{s}$; U(V) = 11.07 $\frac{m}{s}$.

C) V = 338.46(1107) $\frac{m}{s}$.

2.1.5 Value and uncertainty of temperature measurements

The average temperature is:

$$t_{avg} = \frac{24.1 + 24.1 + 24.1}{3} = 24.1°C$$

I consider the **measuring uncertainty** of the measuring instrument to be the smallest measuring plot – **in this case** $0.1°C$. Type A uncertainty assessment is unlikely to make sense here, as the measurement results are constant.

2.1.6 Compliance test of the received value

According to the calculator at the website [2]. We see that the result $V = 338.46\frac{m}{s}$ is quite distant, although not fatal – but definitely not precise. For a temperature of $24.1°C$, an assumed air pressure of 101.325 kPa, and an assumed room humidity of 40%, the sound velocity should be: *346.4* $\frac{m}{s}$. This is a discrepancy close to 7.94 $\frac{m}{s}$. I will use the uncertainty theory to determine whether my calculations and the table result are equal "within the uncertainty of measurement".

We consider the measurement results to be consistent if

$$|x_1 - x_2| < U(x_1 - x_2), \text{ where } U(x_1 - x_2) = k\sqrt{[u(x_1)]^2 + [u(x_2)]^2}$$

Let's assume the calculated velocity is V, and the one from calculator is V_0.

$$|V - V_0| = |338.46\frac{m}{s} - 346.4\frac{m}{s}| = 7.94\frac{m}{s}$$

Then we calculate the extended uncertainty:

$$U(V) = k\cdot u(v) = 2\cdot 11.07\frac{m}{s} = 22.14\frac{m}{s}$$

As $7.94\frac{m}{s} < 22.14\frac{m}{s}$ the result is in accordance with the table value.

2.1.7 An adiabatic index equation

Using the equation:

$$\kappa = \frac{\mu V^2}{RT}, \text{ where } \begin{array}{rl} R = & \text{universal gas constant} \\ \mu = & \text{molar mass of air} \\ T = & \text{air temperature, expressed in K} \end{array}$$

I calculate:

$$\kappa = \frac{\mu V^2}{RT} = \frac{28.97\frac{g}{mol}\cdot\left(338.4615\frac{m}{s}\right)^2}{\left(8.314\ \frac{J}{K\cdot mol}\cdot 297.25\ K\right)} = \frac{3319\frac{kg\cdot m^2}{s^2\cdot mol}}{2471\frac{kg\cdot m^2}{s^2\cdot mol}} = 1.343$$

2.1.8 Calculation of the uncertainty of the obtained adiabate factor

$$u_c(\kappa) = \sqrt{[u(V)]^2\ +\ [u(T)]^2} = \sqrt{\left[\frac{\partial\kappa}{\partial V}\left(\frac{\mu V^2}{RT}\right)u(V)\right]^2 + \left[\frac{\partial\kappa}{\partial T}\left(\frac{\mu V^2}{RT}\right)u(T)\right]^2} =$$

$$= \sqrt{\left[\left(\frac{2\mu V}{RT}\right)\cdot u(V)\right]^2 + \left[\left(-\frac{\mu V^2}{RT^2}\right)\cdot u(T)\right]^2} =$$

$$= \sqrt{\left[\left(\frac{2\cdot 28.97\frac{g}{mol}\cdot 338.4615\frac{m}{s}}{8.314\ \frac{J}{K\cdot mol}\cdot 297.25\ K}\right)\cdot 11.07\frac{m}{s}\right]^2 + \left[\left(-\frac{28.97\frac{g}{mol}\cdot\left(338.4615\frac{m}{s}\right)^2}{8.314\ \frac{J}{K\cdot mol}\cdot(297.25\ K)^2}\right)\cdot 0.1°C\right]^2} =$$

$$= \sqrt{\left[\left(\frac{19.61\frac{kg\cdot m}{s\cdot mol}}{2471\frac{kg\cdot m^2}{s^2\cdot mol}}\right)\cdot 11.07\frac{m}{s}\right]^2 + \left[\left(-\frac{3319\frac{kg\cdot m^2}{s^2\cdot mol}}{734601\frac{kg\cdot m^2\cdot K}{s^2\cdot mol}}\right)\cdot 0.1K\right]^2} = \sqrt{0.007717\ +\ 2.04\cdot 10^{-7}} = 0.08785$$

2.1.9 Testing the compliance of the obtained κ with the table value.

From the table containing the adiabate index of selected gases, I read that for dry air in temperature of $0°C$ the adiabate index κ is *1.403* [3]. In fact, my result is closer to the Cl_2 gas at a temperature of 20°C ($\kappa = 1.340$), than to the dry air itself – which is not exactly what I expected.

Unfortunately the calculated value of κ differs from the table value. Fortunately, the theory of uncertainty comes to the rescue. I will use it to determine whether my measurement is equal to the table value "within the uncertainty of measurement".

Let's assume the calculated index is κ, and the one from table is κ_0.

$$|\kappa - \kappa_0| = |1.343\ -\ 1.403| = 0.06$$

Then we calculate the extended uncertainty:

$$U(\kappa) = k\cdot u(\kappa) = 2\cdot\ 0.08785 = 0.1757$$

We can notice that:

$$|\kappa - \kappa_0| < U(\kappa)$$

This means that we consider the measurement results to be consistent.

2.2 Outside

The second measurement was made outside the house in the late cold and not-windy night, during which the temperature was ~ $2.5°C$ (the temperature was measured three times at about 5 minute intervals). The humidity of that night was ~ 84%. The measured distance was 6 meters.

First device recording [s]	Second device recording [s]	Δt [s]
1.102	1.068	0.034
0.874	0.840	0.034
0.856	0.819	0.037
1.031	0.998	0.033
0.798	0.763	0.035
0.814	0.781	0.033
1.146	1.110	0.036
0.726	0.691	0.035
0.760	0.725	0.035
0.946	0.909	0.037
1.158	1.121	0.037
1.918	1.883	0.035

Figure 4: Measurement of the "claps" timings

Temperature $°C$
2.4
2.5
2.6

Figure 5: Measurement of the temperature

2.2.1 The average value and its uncertainty

From the given data I calculate the average value:

$$\bar{t} = \frac{1}{n}\sum_{i=1}^{n} x_i = \frac{0.034 + 0.034 + 0.037 + \ldots + 0.037 + 0.037 + 0.035}{12} \approx 0.03508333\ [s]$$

From the given data I also calculate the uncertainty of average values:

$$\sigma_{\overline{x}} = \frac{\sigma}{\sqrt{n}} = \sqrt{\frac{1}{n(n-1)}\sum_{i=1}^{n}(t_i - \bar{t})^2} \approx 0.000416667, \quad \text{additionaly: } \sigma = 0.001443376$$

2.2.2 Calculation of sound speed in the air

Using the equation **(1)**:

$$V = \frac{2D}{\Delta t} = \frac{2 \cdot 6\ [m]}{0.035083333\ [s]} = 342.04275\ \frac{[m]}{[s]}$$

Info: We can see that the value is incredibly close to the real value, because for a temperature of $2.5°C$, an assumed air pressure of 101.325 kPa, and a humidity of 84% the speed of sound should be: *333.28* $\frac{m}{s}$. Comparative data was taken from the online calculator [2].

2.2.3 Calculation of the uncertainty of the determined speed

Using the right to propagate uncertainty for the function of many variables I use a formula:

$$u_c(y) = \sqrt{\sum_k \left[\frac{\partial y}{\partial x_k} u(x_k)\right]^2}$$

I consider the measuring uncertainty of the measuring instrument to be the smallest measuring range – 1mm in my case. My measuring tape measured 5 meters and was used to measure 5 meters of distance. In short, the measured distance is 5 meters with an uncertainty of 0.001 meters. Using those informations I calculate:

$$u_c(V) = \sqrt{[u(D)]^2 + [u(t)]^2} = \sqrt{\left[\frac{\partial V}{\partial D}\left(\frac{2D}{t}\right)u(D)\right]^2 + \left[\frac{\partial V}{\partial t}\left(\frac{2D}{t}\right)u(t)\right]^2} = \sqrt{\left[\left(\frac{2}{t}\right)u(D)\right]^2 + \left[\left(-\frac{2d}{t^2}\right)u(t)\right]^2} =$$

$$= \sqrt{\left[\left(\frac{2}{0.03508334\ s}\right)\cdot 0.001m\right]^2 + \left[\left(-\frac{2\cdot 6\ [m]}{(0.03508334\ s)^2}\right)\cdot 0.0004166667\ s\right]^2} = 4.063\ \frac{m}{s}$$

2.2.4 The resulting sound speed value

A) The calculated speed measures 342.043 $\frac{m}{s}$ with an uncertainty of 4.063 $\frac{m}{s}$.

B) V = 342.043 $\frac{m}{s}$; U(V) = 4.063 $\frac{m}{s}$.

C) V = 342.043(4063) $\frac{m}{s}$.

2.2.5 Value and uncertainty of temperature measurements

The average temperature is:

$$t_{avg} = \frac{2.4 + 2.5 + 2.6}{3} = 2.5°C$$

I consider the **measuring uncertainty** of the measuring instrument to be the smallest measuring plot – **in this case** $0.1°C$.

We can also calculate the Type A uncertainty, although I would like to point out here that I only have three measurements of the temperature. According to [4] **for only 3 measurements the uncertainty of assessment is as high as 38%.**

$$\sigma_{t_{avg}} = \frac{\sigma}{\sqrt{n}} = \sqrt{\frac{1}{n(n-1)}\sum_{i=1}^{n}(t_{avg_i} - t_{avg})^2} \approx 0.0577350, \text{ where } \sigma = 0.1$$

2.2.6 Compliance test of the received value

According to the calculator at the website [2] we can see that the value is incredibly close to the real value, because for a temperature of $2.5°C$, an assumed air pressure of 101.325 kPa, and a humidity of 84% the speed of sound should be: *333.28* $\frac{m}{s}$. I will use the uncertainty theory to determine whether my calculations and the table result are equal "within the uncertainty of measurement".

We consider the measurement results to be consistent if

$$|x_1 - x_2| < U(x_1 - x_2), \text{ where } U(x_1 - x_2) = k\sqrt{[u(x_1)]^2 + [u(x_2)]^2}$$

Let's assume the calculated velocity is V, and the one from calculator is V_0.

$$|V - V_0| = 342.043\frac{m}{s} - 333.28\frac{m}{s} = 8,763\frac{m}{s}$$

Then we calculate the extended uncertainty:

$$U(V) = k\cdot u(v) = 2\cdot 4.063\frac{m}{s} = 8.126\frac{m}{s}$$

The resulting value is inconsistent with the value obtained from the calculator ($|V - V_0| \nless U(V)$). The difference V - V_0 is small (in my opinion), which indicates that no gross error was made. There is a chance that the measurement uncertainty has been underestimated – there may have been an additional systematic error in the measurement that could not be detected by statistical analysis of the measurement result.

2.2.7 An adiabatic index equation

Using the equation:

$$\kappa = \frac{\mu V^2}{RT}, \text{ where } \begin{array}{rl} R = & \text{universal gas constant} \\ \mu = & \text{molar mass of air} \\ T = & \text{air temperature, expressed in K} \end{array}$$

I calculate:

$$\kappa = \frac{\mu V^2}{RT} = \frac{28.97\frac{g}{mol}\cdot\left(342.043\frac{m}{s}\right)^2}{\left(8.314\ \frac{J}{K\cdot mol}\cdot 275.65\ K\right)} = \frac{3389\frac{kg\cdot m^2}{s^2\cdot mol}}{2292\frac{kg\cdot m^2}{s^2\cdot mol}} = 1.479$$

2.2.8 Calculation of the uncertainty of the obtained adiabatic index

$$u_c(\kappa) = \sqrt{[u(V)]^2\ +\ [u(T)]^2} = \sqrt{\left[\frac{\partial\kappa}{\partial V}\left(\frac{\mu V^2}{RT}\right)u(V)\right]^2 + \left[\frac{\partial\kappa}{\partial T}\left(\frac{\mu V^2}{RT}\right)u(T)\right]^2} =$$

$$= \sqrt{\left[\left(\frac{2\mu V}{RT}\right)\cdot u(V)\right]^2 + \left[\left(-\frac{\mu V^2}{RT^2}\right)\cdot u(T)\right]^2} =$$

$$= \sqrt{\left[\left(\frac{2\cdot 28.97\frac{g}{mol}\cdot 342.043\frac{m}{s}}{8.314\ \frac{J}{K\cdot mol}\cdot 275.65\ K}\right)\cdot 4.063\frac{m}{s}\right]^2 + \left[\left(-\frac{28.97\frac{g}{mol}\cdot\left(342.043\frac{m}{s}\right)^2}{8.314\ \frac{J}{K\cdot mol}\cdot(275.65\ K)^2}\right)\cdot 0.1°C\right]^2} =$$

$$= \sqrt{\left[\left(\frac{19.82\frac{kg\cdot m}{s\cdot mol}}{2292\frac{kg\cdot m^2}{s^2\cdot mol}}\right)\cdot 4.063\frac{m}{s}\right]^2 + \left[\left(-\frac{3389\frac{kg\cdot m^2}{s^2\cdot mol}}{631705\frac{kg\cdot m^2\cdot K}{s^2\cdot mol}}\right)\cdot 0.1K\right]^2} = \sqrt{0.001234\ +\ 2.88\cdot 10^{-7}} = 0.03514$$

A) The calculated adiabatic index measures 1.479 with an uncertainty of 0.035.

B) κ = 1.479; U(κ) = 0.035.

C) κ = 1.479(35).

2.2.9 Testing the compliance of the obtained κ with the table value.

From the table containing the adiabate index of selected gases, I read that for dry air and temperature of $0°C$ the adiabate index κ is *1.403* [3].

Unfortunately the calculated value of κ differs from the table value. Fortunately, the theory of uncertainty comes to the rescue.

Let's assume the calculated index is κ, and the one from table is κ_0.

$$|\kappa - \kappa_0| = |1.479\ -\ 1.403| = 0.076$$

Then we calculate the extended uncertainty:

$$U(\kappa) = k\cdot u(\kappa) = 2\cdot 0.03514 = 0.07028$$

We can notice that:

$$|\kappa - \kappa_0| > U(\kappa)$$

Well, unfortunately, this time the obtained value is not consistent with the table value. Rather, the measurement uncertainties were assessed too low.

3 Conclusions

I think that the measurements I obtained are satisfactory. For one measurement I obtained results consistent with the tabular value – and for the other I did not, although they were not gross errors. What shocked me the most was that I was most satisfied with the second measurement (*2.2. Outside*). However, I almost certainly think that the underestimated uncertainty (only ~ $4\ m/s$) failed here. I think I should have taken more measurements to select and throw away the strongly outliers. I did this for the first measurement, and for the second measurement a frosty night quickly blew me back home.

An additional obstacle on the outside was that the device with phyphox application liked to cooperate much less. Many of the hand-claps had to be repeated in order for the acoustic stopwatch to register them. If I had to take these measurements again, I would have used something much louder as a sound - like a whistle, which I did not have at the time.

References

[1] Sarah Maddison. Sound - Part 1 — Sarah's Scientists in Schools Projects. `http://astronomy.swin.edu.au/~smaddiso/astro/SiS/NPS/sound1.html`. [Online; accessed 3-December-2020].

[2] Alexander Sengpiel. Tontechnik-Rechner - segnpielaudio. `http://www.sengpielaudio.com/calculator-airpressure.htm`. [Online; accessed 6-December-2020].

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[4] Andrzej Zięba. Opracowanie danych pomiarowych. Chapter 1.